NORTHERN BEACHES SECONDARY COLLEGE

MANLY SELECTIVE CAMPUS

HIGHER SCHOOL CERTIFICATE

TRIAL EXAMINATION

2019

Mathematics Extension II

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black pen
- Write your Student Number at the top of each page
- Answer Section I- Multiple Choice on Answer Sheet provided
- Answer Section II – Free Response in a separate booklet for each question.
- NESA approved calculators and templates may be used.

Section I Multiple Choice
- 10 marks
- Attempt all questions
- Allow about 15 minutes for this section

Section II – Free Response
- 90 marks
- Each question is of equal value
- All necessary working should be shown in every question.
- Allow about 2 hours 45 minutes for this section

Weighting: 30%

Section 1: Multiple Choice (10 marks)

Indicate your answer on answer sheet provided.

Allow approximately 15 minutes for this section.

Q1. A mass of 5 kg moves in a horizontal circle of radius 1.5 metres at a uniform angular speed of 4 radians per second. What is the centripetal force required for this motion?

A. 40N

B. 80N

C. 120N

D. 160N

Q2. Find $\int \frac{dx}{x^2 - 6x + 13}$

A. $\frac{1}{2}\tan^{-1}\left(\frac{x-3}{2}\right) + C$

B. $\frac{1}{3}\tan^{-1}\left(\frac{x+3}{2}\right) + C$

C. $\frac{1}{2}\tan^{-1}\left(\frac{x-2}{3}\right) + C$

D. $\frac{1}{3}\tan^{-1}\left(\frac{x+2}{3}\right) + C$

Q3. What is the equation to the chord of contact to the ellipse $9x^2 + 16y^2 = 144$ from the point (8,6)?

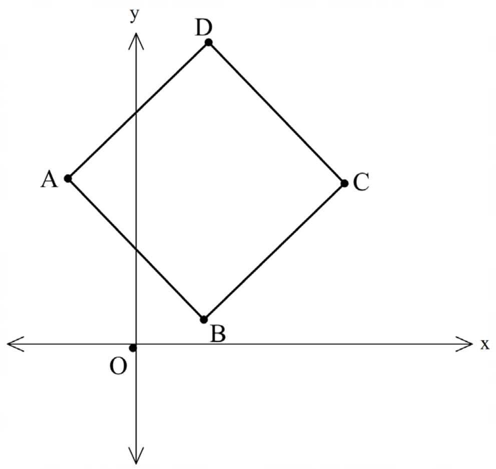
A. $3x + 4y = 6$

B. $3x + 6y = 2$

C. $9x + 16y = 144$

D. $6x + 8y = 12$

Q4. In the Argand diagram $ABCD$ is a square and the vertices A and B correspond the complex numbers ω and z .



Which complex number corresponds to the diagonal BD?

A. $(\omega - z)(1 - i)$

B. $(\omega - z)(1 + i)$

C. $(z - \omega)(1 + i)$

D. $(\omega + z)(1 - i)$

Q5. The polynomial $P(x) = x^3 + 2x^2 - 5x + 7$ has roots α, β and γ.
Which polynomial has roots $\alpha + 1$, $\beta + 1$ and $\gamma + 1$?

A. $x^3 - x^2 + 6x + 13 = 0$

B. $x^3 - x^2 - 6x + 13 = 0$

C. $x^3 - x^2 - 6x - 13 = 0$

D. $x^3 + x^2 - 6x - 13 = 0$

Q6. What is an expression for the constant B such that $P(x) = (x - \alpha)^2 Q(x) + Ax + B$?

A. $B = P(\alpha)$

B. $B = P`(\alpha)$

C. $B = P`(\alpha) - \alpha P(\alpha)$

D. $B = P(\alpha) - \alpha P`(\alpha)$

Q7. Let ω be a complex cube root of -1. The value of $(1 + \omega - \omega^2)^3$ is:

A. 1

B. -1

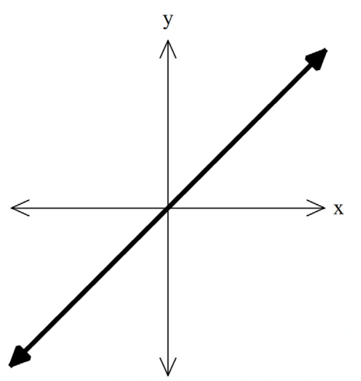
C. 8

D. -8

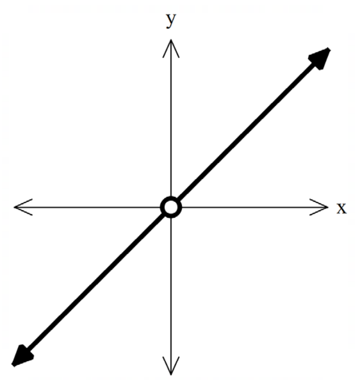
Q8. The equation $\frac{x}{y} + \frac{y}{x} = 2$ is an implicit function in x and y.

Which graph represents this implicit function?

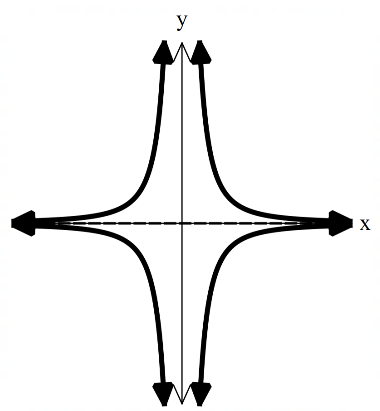
A



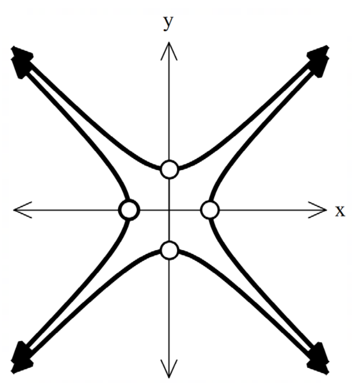
B



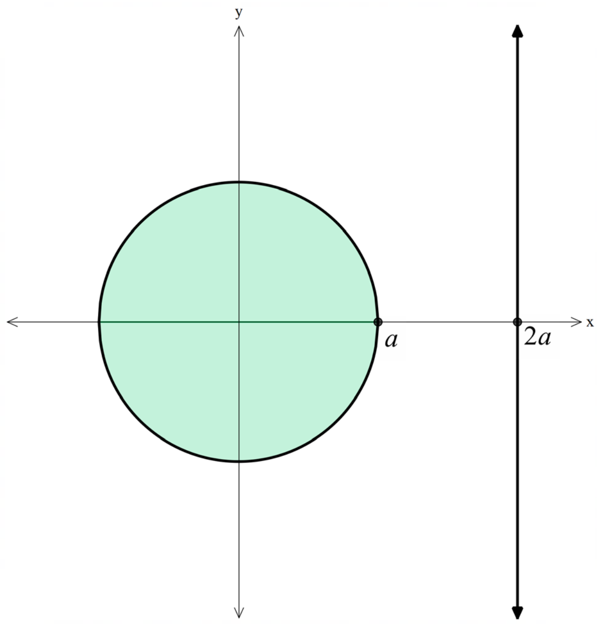
C



D



Q9. The diagram below shows the circle $x^2 + y^2 = a^2$.



Solid A is formed by rotating the area enclosed by the circle around the line $x = 2a$.

The volume of solid A is V_A

Another solid, Solid B, is formed by rotating the area enclosed by the circle around the line $x = 4a$.

The volume of solid B is V_B

Which of the following gives the correct volume of solid B?

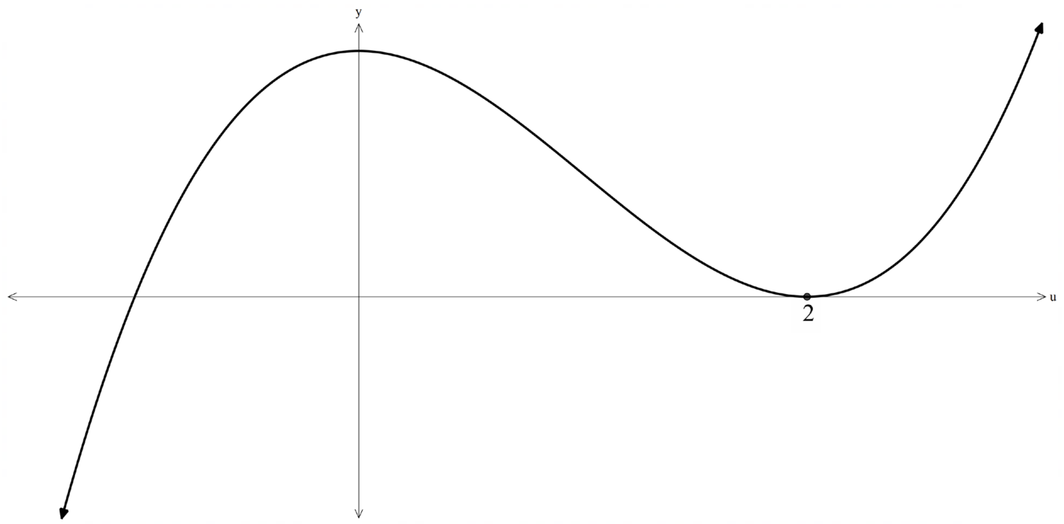
A. $V_B = 2V_A$

B. $V_B = 4V_A$

C. $V_B = 8V_A$

D. $V_B = 16V_A$

Q10. The graph below shows $y = f(u)$.



The function $g(x)$ is defined as $g(x)\int_0^x f(u)\ du$.

Which of the statements below is true?

A. $g(0) = 0$ and $g'(0) = 0$

B. $g(0) > 0$ and $g'(2) = 0$

C. $g''(0) = 0$ and $g'(2) = 0$

D. $g'' > 0$ and $g'(2) = 0$

<u>End of Multiple Choice</u>

Section II Total Marks is 90

Attempt Questions 11 – 16.

Allow approximately 2 hours & 45 minutes for this section.

Answer all questions, starting each new question in a new booklet.

All necessary working must be shown in each and every question.

Question 11. – Start New Booklet **15 marks**

a. i Express $z = 2 - 2\sqrt{3}\, i$ in modulus-argument form **2**

ii Hence, otherwise, evaluate z^5 in simplest Cartesian form. **1**

b. Draw on an Argand diagram, the region defined by $Im(2z + iz) \geq 2$ **2**

[Your diagram must be at least one third of a page in size and must be **<u>neat</u>** and fully labelled.]

c. Let $x = \alpha$ be a root of the polynomial $P(x) = x^4 + Ax^3 + Bx^2 + Ax + 1$

where A and B are real numbers and $4A^2 \neq (2 + B)^2$

i. Show that α cannot be 0, 1 or -1. **3**

ii. Show that $P\left(\dfrac{1}{\alpha}\right) = 0$ **1**

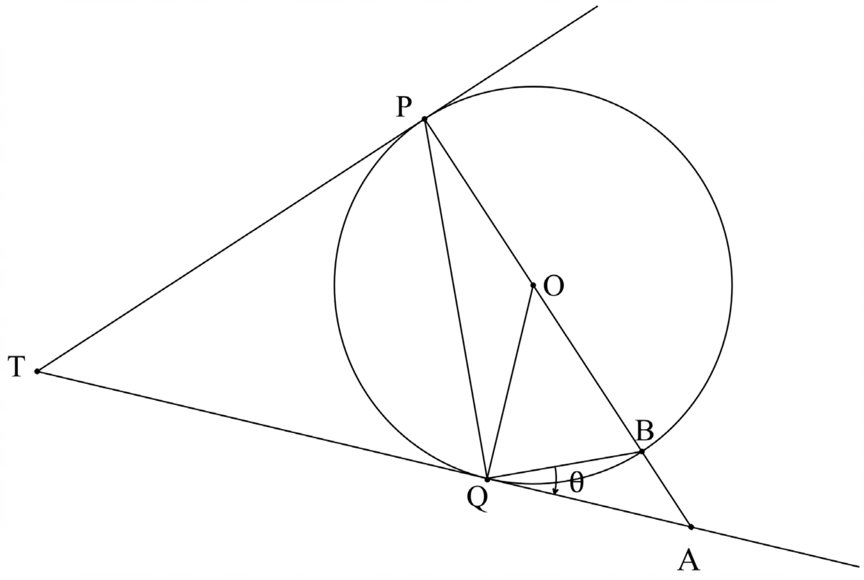
Question 11 continues on the next page.

Question 11 continued.

d. From an external point T, tangents are drawn to a circle with centre O, touching the circle at P and Q. $\angle PTQ$ is acute.

The diameter PB produced meets the tangent TQ at A.

Let $\angle AQB = \theta$



(The diagram has been reproduced in your answer booklet. Answer this question on that page)

i) Show that $\angle PTQ = 2\theta$ **2**

ii) Prove that $\Delta PBQ \parallel\!| \Delta TOQ$ **2**

iii) Hence, show that $BQ.OT = 2(OP)^2$ **2**

End of Question 11

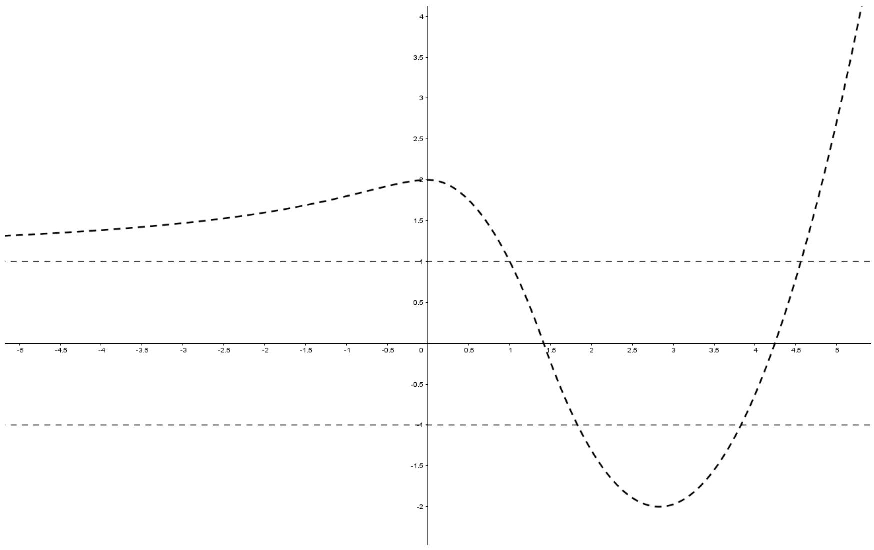
Question 12. – Start New Booklet **15 marks**

a. A vehicle of mass 3000 kg is travelling around a horizontal circular road of radius 100*m* at a speed of 7.5 *m/sec*. Determine the centripetal force acting on the vehicle. **2**

b. i Write $\dfrac{2x^2+3x-3}{x^2-1}$ in the form $A+\dfrac{B}{x-1}+\dfrac{C}{x+1}$. **2**

ii Hence find $\displaystyle\int \frac{2x^2+3x-3}{x^2-1}\,dx$. **1**

c. The diagram shows the curve $f(x)$. The curve $f(x)$ is asymptotic to $y = 1$. The y-intercept is (0,2).



This curve *f(x)* has been reproduced in your answer booklet.
Sketch the following curves showing all intercepts and asymptotes.

i) $y = f(|x|)$ **1**

ii) $y = \sqrt{f(x)}$ **2**

iii) $y = \dfrac{1}{f(x)}$ **2**

iv) $|y| = f(x)$ **2**

v) $y = \ln\,[f(x)]$ **3**

End of Question 12

Question 13. – Start New Booklet **15 marks**

a. Let $P(x) = x^4 + mx^3 + 36x^2 - 35x + n$ where m and n are real numbers.

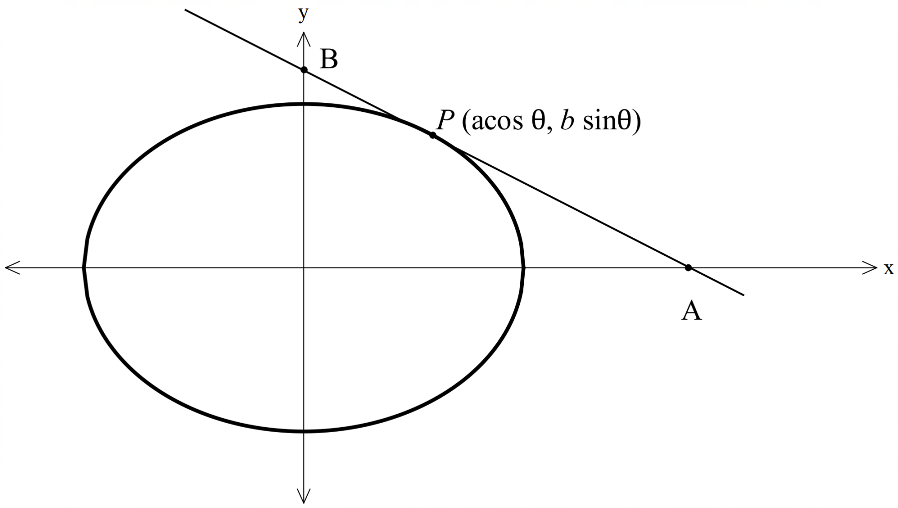
It is given that $P(5) = 0$ and $P\left(\dfrac{1-\sqrt{3}\,i}{2}\right) = 0$.

i) Show that $x^2 - x + 1$ is a factor of $P(x)$. **2**

ii) Find m and n. **2**

b. In the diagram below, $P\,(\,a\cos\Theta\,,\,b\sin\Theta)$ lies on the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$

The tangent at P cuts the x-axis at A and the y-axis at B.



i. Derive the parametric equation of the tangent at P in any form, and find the coordinates of A and B in parametric form. **2**

ii. Show that $\dfrac{PA}{PB} = \tan^2\theta$ **2**

c. A string is 0.5m long and will break if an object of mass exceeding 40kg is hung vertically from it. An object of mass 2kg is attached to one end of the string and it revolves around a horizontal circle with uniform speed. (Let gravity $g = 9.8\ m/\text{sec}^2$)

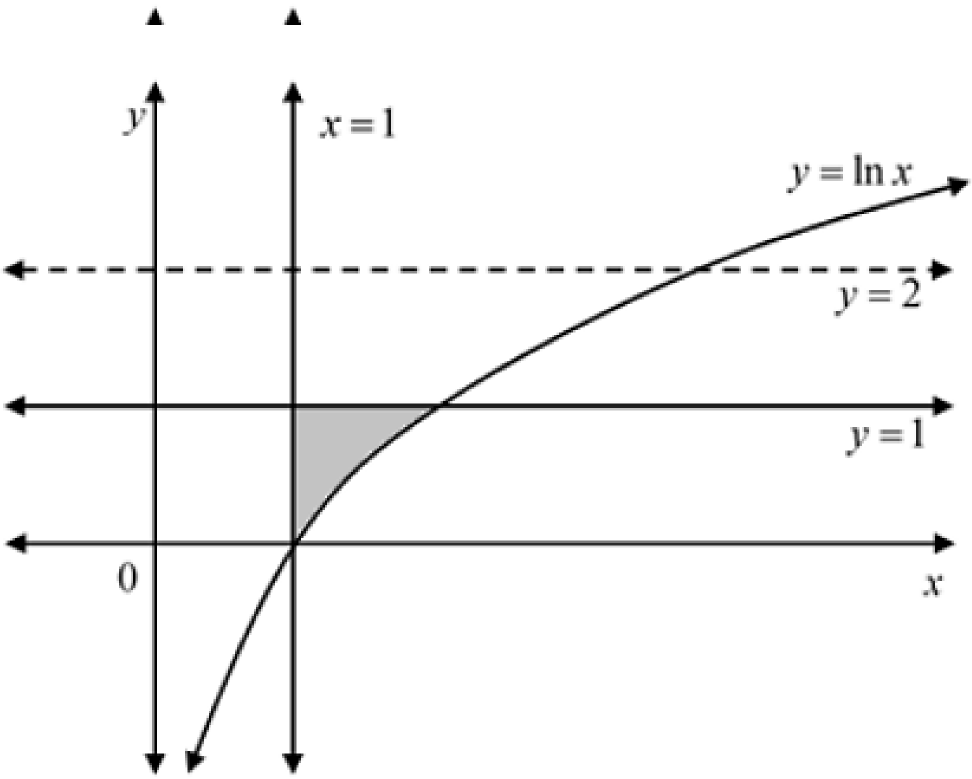
i. Find the greatest angular velocity which may be imparted to the object without breaking the string **2**

ii. Find the tangential speed at which this occurs. **1**

Question 13 continues on the next page.

Question 13 continued. **Marks**

d. The region bounded by the curve y = ln x, $x = 1$ and $y = 1$ is shaded in the diagram below. The region is rotated about the line $y = 2$ to form a solid.



Find the volume of the solid formed using the method of cylindrical shells. 4

End of Question 13

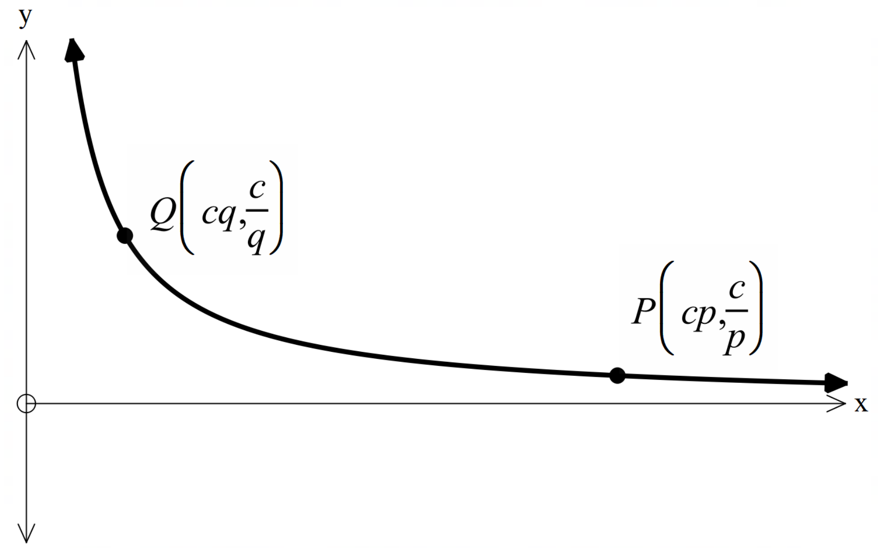
Question 14. – Start New Booklet **15 marks**

a. Given $t = \tan x$,

i. Show that $\dfrac{dx}{dt} = \dfrac{1}{1+t^2}$. **1**

ii Use the substitution $t = \tan x$ to find $\displaystyle\int \frac{dx}{1 + \sin 2x}$ **3**

b. The variable points $P\left(cp, \dfrac{c}{p}\right)$ and $Q\left(cq, \dfrac{c}{q}\right)$, where $p > q > 0$ lie on the hyperbola $xy = c^2$. M is the midpoint of PQ.

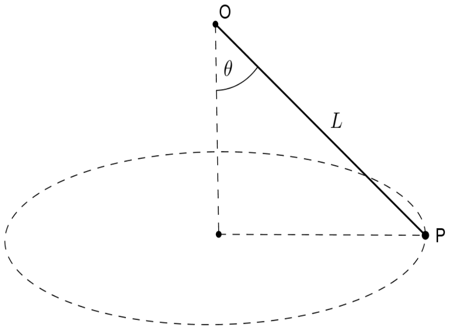


Given $p - q = 4$, find the equation of the locus of M. **3**

Question 14 continues on next page.

Question 14 continued

c. The diagram below shows a particle P of mass M kilograms suspended from a fixed point O by an inextensible string of length L metres.



P moves in a circle with centre directly below and distance h from O with uniform angular speed ω radians/sec.

The string makes an angle θ with the vertical and the acceleration due to gravity is $g\ ms^{-2}$.

i. Prove that the period of this motion is $2\pi\sqrt{\frac{h}{g}}$ 2

ii. By considering the forces acting on the particle show that $\cos\theta = \dfrac{g}{L\omega^2}$. 3

iii. The angular speed of the particle is increased to μ radians/sec.
At that speed the string makes an angle 2θ with the vertical.

Show that $\mu^2 = \dfrac{gL\omega^4}{2g^2 - L^2\omega^4}$. 3

End of Question 14

Question 15. – Start New Booklet **15 marks**

a. Find

i $\int x e^{2x}\ dx$ **3**

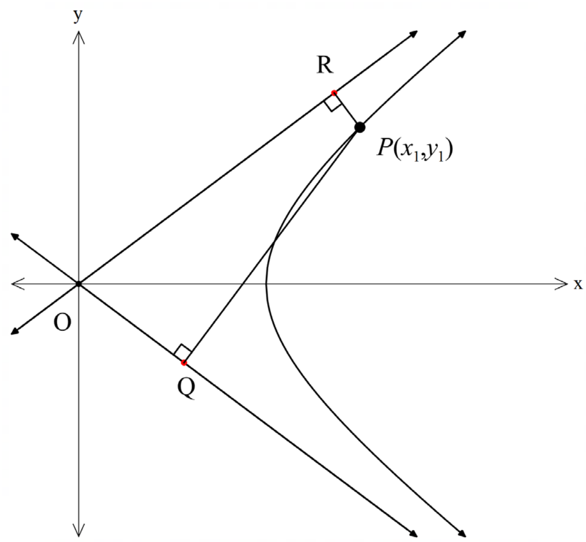
ii $\int_0^{\frac{\pi}{2}} \sin\theta\ (1-\cos\theta)^2\ d\theta$ **3**

b. i. Find the non-real solutions for of the equation $z^7 - 1 = 0$ **2**

ii Express $z^7 - 1$ as a product of linear and quadratic factors with real coefficients. **2**

iii Hence prove that $\cos\frac{\pi}{7} + \cos\frac{3\pi}{7} + \cos\frac{5\pi}{7} = \frac{1}{2}$ **2**

c. The diagram below shows the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

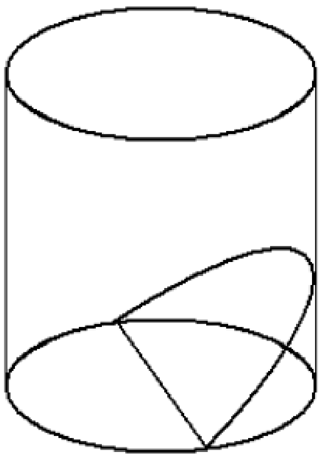


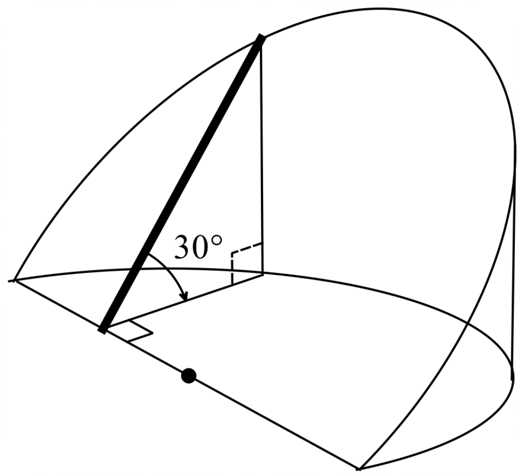
The point $P(x_1, y_1)$ is a point on the hyperbola.
The points Q and R lie on the asymptotes of the hyperbola such that $\angle PQO = \angle PRO = 90^\circ$. The eccentricity of the hyperbola is *e*.

Show that $PQ \times PR = \frac{b^2}{e^2}$. **3**

End of Question 15

Question 16. – Start New Booklet **15 marks**





a. A wedge is cut out of a circular cylinder of radius 5 cm. One plane is perpendicular to the axis of the cylinder. The other intersects the first plane at an angle of 30° along the diameter of the cylinder.

The cross section is a triangle with its base perpendicular to the diameter.

Find the volume of the shape. **4**

$$\int \frac{\ln(1+x)}{1+x^2}\,dx = \int \ln(1+\tan\theta)\,d\theta$$

let $x = \tan\theta \Rightarrow dx = \sec^2\theta$

$$\therefore \qquad I = \int \frac{\ln(1+\tan x)}{1+\tan^2 x}\sec^2\theta\,d\theta$$

$$= \int \frac{\ln(1+\tan x)}{\sec^2\theta}\sec^2\theta\,d\theta$$

b. i Show $\displaystyle = \int \ln(1+\tan\theta)\,d\theta$ **1**

ii Hence, or otherwise, evaluate $\displaystyle\int_0^1 \frac{\ln(1+x)}{1+x^2}\,dx$ **3**

c. The integral I_n is defined as $I_n = \int_0^1 \frac{1}{(1+x^2)^n}\,dx$ (for integers $n \geq 1$).

i. Show that $\int \frac{x}{(1+x^2)^n}\,dx = \frac{-1}{2(n-1)} \times \frac{1}{(1+x^2)^{n-1}} + C$ **2**

ii. By considering $\frac{1}{(1+x^2)^n} = \frac{1+x^2}{(1+x^2)^n} - \frac{x^2}{(1+x^2)^n}$, or otherwise,

show that $I_n = \frac{2n-3}{2(n-1)} I_{n-1} + \frac{1}{(n-1)\times 2^n}$ for $n \geq 2$. **3**

iii. Show that $I_n > \frac{1}{2^n}$ for $n \geq 1$. **2**

End of Examination

Multiple Choice

Q1. C Q2. A Q3.A Q4.A Q5.B Q6D* Q7C Q8B Q9A Q10.C

Q1	$F = mr\omega^2$ $= 5 \times 1.5 \times 4^2$ $= 120N$	C
Q2	$\int \frac{dx}{x^2 - 6x + 13}$ $= \int \frac{dx}{x^2 - 6x + 9 + 4}$ $= \int \frac{dx}{(x-3)^2 + 2^2}$ $= \frac{1}{2}\tan^{-1}\frac{x-3}{2} + C$	A
Q3	$9x^2 + 16y^2 = 144$ $\frac{x^2}{16} + \frac{y^2}{9} = 1$ $\frac{x x_0}{16} + \frac{y y_0}{9} = 1$ $\frac{8x}{16} + \frac{6y}{9} = 1$ $\frac{x}{2} + \frac{2y}{3} = 1$ $3x + 4y = 6$	A
Q4	$\overrightarrow{BD} = \overrightarrow{BA} + \overrightarrow{AD}$ $\overrightarrow{BA} = \omega - z$ $\overrightarrow{AB} = z - \omega$ $\overrightarrow{AD} = i\overrightarrow{AB} = i(z-\omega)$ $\overrightarrow{BD} = (\omega - z) + i(z - \omega)$ $= (\omega - z)(i - i)$	A

Q5	$X = x + 1 \Rightarrow X - 1$ $(X-1)^3 + 2(X-1)^2 - 5(X-1) + 7$ $= X^3 - 3X^2 + 3X - 1 + 2X^2 - 4X + 2$ $-5X + 5 + 7$ $= X^3 - X^2 - 6X + 13$ $= x^3 - x^2 - 6x + 13$	B
Q6	$P(x) = (x-\alpha)^2 Q(x) + Ax + B$ let $x = \alpha$ $P(\alpha) = A\alpha + B$ $P'(x) = 2(x-\alpha)Q(x) + (x-\alpha)^2 Q'(x) + A$ $P'(\alpha) = A$ $P(\alpha) - \alpha P'(\alpha) = A\alpha + B - \alpha A$ $B = P(\alpha) - \alpha P'(\alpha)$	D
Q7	$z^3 + 1 = 0$ $(z+1)(1 - z + z^2) = 0$ if ω is a complex root $\therefore$ $1 - \omega + \omega^2 = 0$ $1 - \omega = -\omega^2$ $(1 + \omega - \omega^2)^3$ $= (1 + \omega + 1 - \omega)^3$ $= (2)^3$ $= 8$	C
Q8	$\frac{x}{y} + \frac{y}{x} = 2$ $x^2 + y^2 = 2xy$ $x^2 - 2xy + y^2 = 0$ $(x-y)^2 = 0$ $y = x$ $x \neq 0; y \neq 0$	B

Q9

Short solution

Taking slices of width Δy perpendicular to axis of rotation:

$$V_A = \pi\int_{-a}^{a}\left[(2a+x)^2-(2a-x)^2\right]dy$$

$$= \pi\int_{-a}^{a}(8ax)\,dy$$

$$= 4\pi^2a^3 \ units^3 \ (\text{given})$$

$$V_B = \pi\int_{-a}^{a}\left[(4a+x)^2-(4a-x)^2\right]dy$$

$$= \pi\int_{-a}^{a}(16ax)\,dy$$

$$= 2\times V_A$$

$$= 8\pi^2a^3 \ units^3$$

Long solution

Voume = $\pi\,(R^2-r^2)$ height

$= \pi\,(R+r)(R-r)\,\delta y$

$R = 4a + x$

$r = 4a - x$

$V = \pi\ 8a \times 2x\delta y$

$$= 16\pi a\int_{-a}^{a} x\,dy$$

$$= 2\times 16\pi a\int_{0}^{a}\sqrt{a^2-y^2}\,dy$$

$$= 32\pi a\int_{0}^{a}\sqrt{a^2-y^2}\ dy$$

let $y = a\sin\theta$ $\ dy = a\cos\theta$

$y = a \Rightarrow \ a\sin\theta = a \Rightarrow \ \theta = \dfrac{\pi}{2}$

$$= 32\pi a\int_{0}^{\frac{\pi}{2}} a\cos\theta a\cos\theta\ d\theta$$

$$= 32\pi a^3\int_{0}^{\frac{\pi}{2}}\frac{1}{2}(\cos 2\theta + 1)\ d\theta$$

$$= 16\pi\, a^3\left[\frac{1}{2}\sin 2\theta + \theta\right]_0^{\frac{\pi}{2}}$$

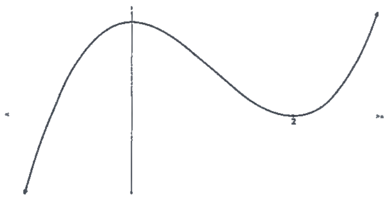
$$= 16\pi a^3\left\{\left(\frac{1}{2}\sin\pi + \frac{\pi}{2}\right) - (0+0)\right\}$$

$$= 8\pi^2a^3$$

A

MSC HSC Mathematics X2 Solutions

Q10



$g''(0)=0$ and $g'(2)=0$

Given

$$g(x) = \int_0^x f(x)\ dx$$

then $f(x) = g'(x)$

$\therefore$ $f'(x) = g''(x)$

From graph

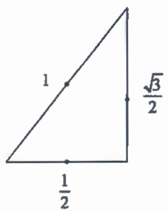
$f'(0) = 0 \therefore g''(0) = 0$

Similarly as $f(x) = g'(x)$

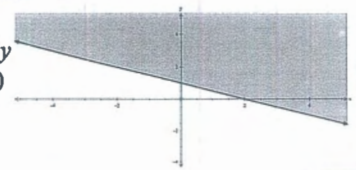
$\therefore$ $f(2) = g'(2) = 0$

Option C

C

a-i	*m* and *n* are real therefore roots are in conjugate pairs. Roots are $5, \left(\frac{1-\sqrt{3}\,i}{2}\right), \left(\frac{1+\sqrt{3}\,i}{2}\right)$ Factors $(x-5)\left(x-\frac{1-\sqrt{3}\,i}{2}\right), \left(x-\frac{(1)+\sqrt{3}\,i}{2}\right)$ $(x-z)(x-\bar{z}) = x^2 - 2Re(z) + (\lvert z\rvert)^2$ $\left(x-\frac{1-\sqrt{3}\,i}{2}\right)\left(x-\frac{(1)+\sqrt{3}\,i}{2}\right)$ $= \left(x^2 - 2\times\frac{1}{2}\times x + 1\right)$ 	2 marks – correct solution

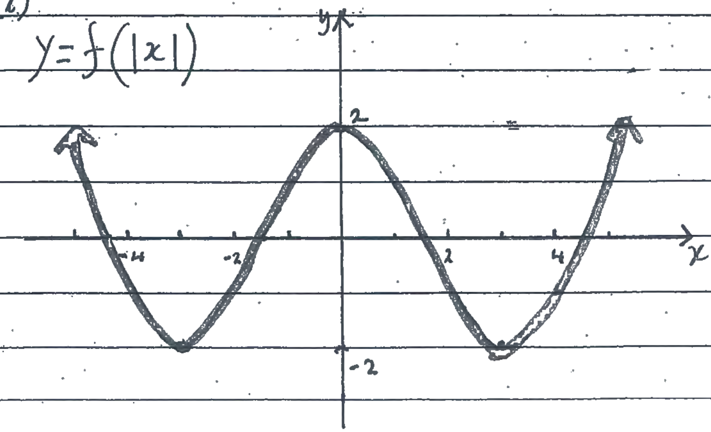
MSC HSC Mathematics X2 Solutions
Question 11

ai	$\lvert z\rvert = \sqrt{2^2 = (-2\sqrt{3})^2}$ $= \sqrt{16}$ $= 4$ $Arg(z) = -\tan^{-1}\left(\frac{2\sqrt{3}}{2}\right)$ $= -\tan^{-1}(\sqrt{3})$ $= -\frac{\pi}{3}$ $\therefore \quad z = 4cis\left(-\frac{\pi}{3}\right)$	2 marks-correct solution 1 mark-correct mod or arg
aii	$z^5 = \left[4cis\left(-\frac{\pi}{3}\right)\right]^5$ $= 4^5 cis\left(-\frac{5\pi}{3}\right)$ $= 1024\left(\cos\left(-\frac{5\pi}{3}\right) + i\sin\left(-\frac{5\pi}{3}\right)\right)$ $= 1024\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$ $= 1024\left(\frac{1}{2} + i\sqrt{\frac{3}{2}}\right)$ $= 512 + 512\sqrt{3}\,i$	1 mark-correct answer
b	let $z = x + iy$ $2z + iz = 2x + 2iy + iy + i^2y$ $= 2x - y + i(x + 2y)$ $Im(2z + z) = x + 2y$ $\therefore \quad x + 2y \geq 2$ 	2 marks-correct solution 1 mark-correct inequality but incorrect region
ci	$P(0) = 1 \therefore \alpha \neq 0$ $P(1) = 2 + 2A + B$ if $\alpha = 1$ then $P(1) = 0$ then $2 + B = -2A$ $(2+B)^2 = (-2A)^2$ $(2+B)^2 = 4A^2$ but $(2+B)^2 \neq 4A^2$ $\therefore \quad \alpha \neq 1$ $P(-1) = 2 - 2A + B$ if $\alpha = -1$ then $P(-1) = 0$ then $2 + B = 2A$ $(2+B)^2 = (2A)^2$ $(2+B)^2 = 4A^2$ but $(2+B)^2 \neq 4A^2$ $\therefore \quad \alpha \neq -1$	3 marks –correct solution 2 marks-ONLY one error in correct progress to proof 1 mark- ONLY one correct root shown

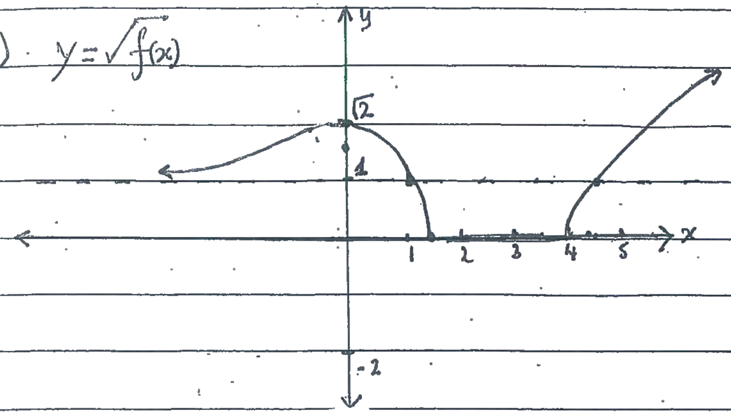
cii	$P\left(\frac{1}{\alpha}\right)=\left(\frac{1}{\alpha}\right)^4+A\left(\frac{1}{\alpha}\right)^3+B\left(\frac{1}{\alpha}\right)^2+A\left(\frac{1}{\alpha}\right)+1$ $=\frac{1}{\alpha^4}+\frac{A}{\alpha^3}+\frac{B}{\alpha^2}+\frac{A}{\alpha}+1$ $=\frac{1}{\alpha^4}(A\alpha+B\alpha^2+A\alpha^3+\alpha^4)$ and $P(\alpha)=0$ ie $A\alpha+B\alpha^2+A\alpha^3+\alpha^4=0$ $\therefore P\left(\frac{1}{\alpha}\right)=\frac{1}{\alpha^4}\times 0$ $=0$	1 mark correct solution
di	$\angle BPQ=\theta$(alternate segment) $\angle BOQ=2\theta$($\angle$ centre is $2\times\angle$ at circumference) $\angle POQ+\angle BOQ=180°$(supplementary) $\angle TPO=\angle TQO=90°$(tangent $\perp$ radii) $\angle PTQ+\angle tPOQ=180°$($\angle$ sum Δ) $\therefore\ \angle PTQ=\angle BOQ=2\theta$	2 marks-correct solution 1 mark- one correctly used relevant theorem in a progress to proof.
dii	$\Delta PBQ\ \Delta TOQ$ $\angle PQB=90°$($\angle$ in semicircle) $\angle OQT=90°$(tangent $\perp$ radii) $\therefore\ \angle PQB=\angle OQT$ ΔPOQ isosceles(equal radii) $\angle TOQ=\frac{180-2\theta}{2}$ (base $\angle$ of Isos Δ) $=90-\theta$ $\angle PBQ=90-\theta$($\angle$ sum Δ) $\therefore\ \angle TOQ=\angle PBQ$ $\therefore\ \Delta PBQ\,\|\|\,\Delta TOQ$ (equiangular)	2 marks-correct solution 1 mark- one correctly used relevant theorem in a progress to proof.
diii	$\frac{OT}{BP}=\frac{OQ}{BQ}$(corresponding sides in equal rat*ι*) $BP=OP+OB$ (diameter-given) $OP=OB=OQ$ (equα radii) $\therefore\ BP=2OP$ hence $\frac{OT}{2OP}=\frac{OP}{BQ}$ $BQ.OT=2OP.OP$ $=2(OP)^2$	2 marks-correct solution 1 mark- one correctly used relevant theorem in a progress to proof.

Question 12

	Marks
(a) $F_c=\frac{mv^2}{r}\Rightarrow F_c=\frac{3000\times 7.5^2}{100}$	2 Marks Correct solution
$=1687.5\ N$	1 Mark Correct substitution
(b) (i) $\frac{2x^2+3x-3}{x^2-1}=A+\frac{B}{x-1}+\frac{C}{x+1}$	2 Marks Correct solution
$2x^2+3x-3=A(x^2-1)+B(x+1)+C(x-1)$	
$x=1\Rightarrow B=1$; $x=-1\Rightarrow C=2$; $x=0\Rightarrow A=2$	1 Mark Obtains one correct value for A or B or C
$\frac{2x^2+3x-3}{x^2-1}=2+\frac{1}{x-1}+\frac{2}{x+1}$	
(ii) $\int\frac{2x^2+3x-3}{x^2-1}\,dx=\int\left\{2+\frac{1}{x-1}+\frac{2}{x+1}\right\}dx$	1 Mark Correct solution
$=2x+\log_e(x-1)+2\log_e(x+1)$	
(c) (i) $y=f(\|x\|)$	1 Mark Correct graph



(ii) $y=\sqrt{f(x)}$



Marks

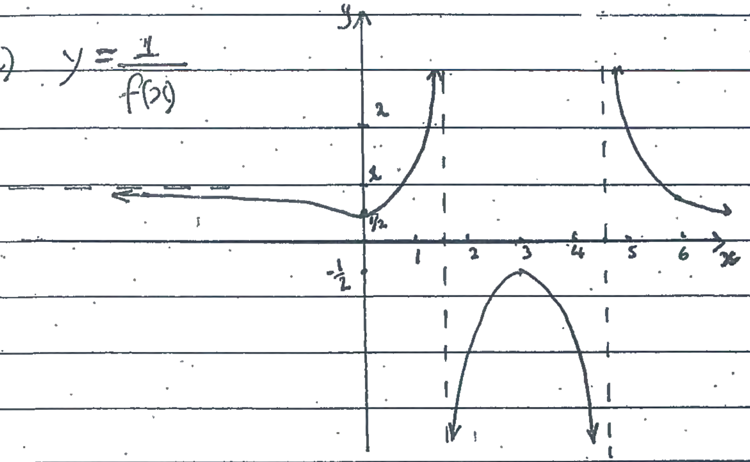
2 Marks

Correct graph

1 Mark

Correct LHS including y intercept, explicit or implicit.

(iii) $y=\dfrac{1}{f(x)}$



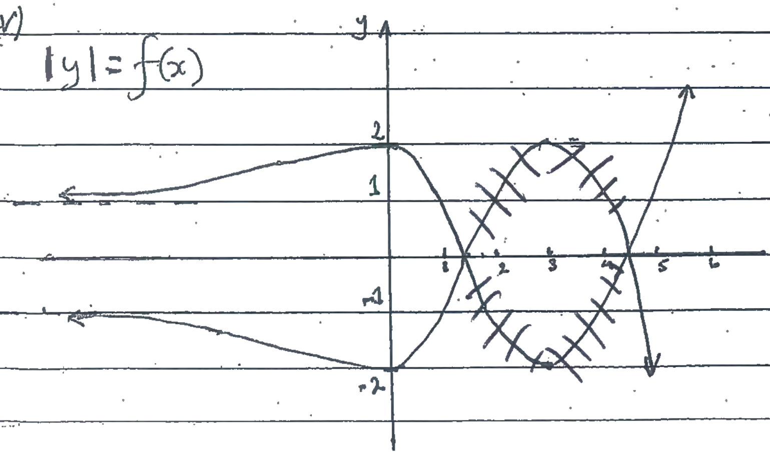
2 Marks

Correct graph

1 Mark

Two of three features correct

(iv) $|y| = f(x)$



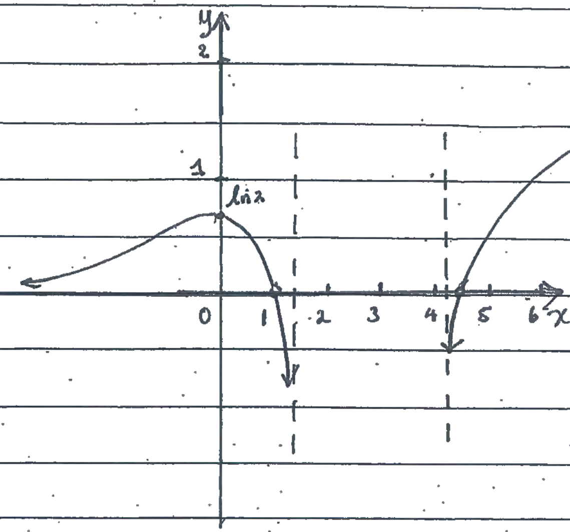
2 Marks

Correct graph

1 Mark

Correct ~~LHS and RHS but exclud graph for 1.5≤x≤4.5~~

(v)



Marks

3 Marks

Correct graph

2 Marks

Graph with two of three asymptotes correct AND correct y intercept, explicit or implicit

1 Mark

Two correct components other than asymptotic behaviour of graph a-ii

$$P(x) = (x-5)(x^2 - x + 1)(x + a)$$

$x = 0$
$n = -5a$

$x = 1$
$1 + m + 36 - 35 + n = -4 - 4a$
$m + n + 2 = -4 - 4a$
$m - 5a = -6 - 4a$
$m - a = -6$
$m = a - 6$

$x = -1$
$1 - m + 36 + 35 + n = -6 \times 3 \times (a-1)$
$n - m + 72 = 18 - 18a$
$-5a - m + 18a = -54$
$13a - (a-6) = -54$
$12a + 6 = -54$
$12a = -60$
$a = -5$

$\therefore \quad m = -11 \qquad n = 25$

Question 13

a

$$P(x) = x^4 + mx^3 + 36x^2 - 35x + n$$
$$P(x) = (x-5)(x-z)(-\overline{z})(x^2 + ax + b)$$
$$(x-z)(x-\overline{z}) = x^2 - 2Re(z) + (|z|)^2$$
$$z = \frac{1}{2} - \frac{\sqrt{3}}{2}i$$
$$|z| = \sqrt{\frac{1}{4} + \frac{3}{4}} = 1$$
$$p(x) = (x-5)\left(x^2 - 2 \times \frac{1}{2}x + (1)^2\right)(ax + b)$$
$$P(x) = (x-5)(x^2 - x + 1)(x + b)$$

Method 1. Substitution.

Let $x = 1$
$LHS = 1 \times -4 \times (1 - \beta) = 4\beta - 4$
$RHS = 1 + m + 36 - 35 + n$
$\therefore \quad 4\beta - 6 = m + n$ ①

Let $x = 0$
$LHS = 5\beta$
$RHS = n$
$\therefore \quad 5\beta = n$ ②

Let $x = 1$
$LHS = 3 \times -6 \times (-1 - \beta)$
$= 18 + 18\beta$
$RHS = 1 - m + 36 + 35 + n$
$= 72 + n - m$
$\therefore \quad 18\beta - 54 = n - m$ ③

① + ③
$22\beta - 60 = 2n$
Subst ②
$22b^2 - 60 = 10\beta$
$12\beta = 60$
$\beta = 5$

$\therefore \quad n = 5\beta = 25$
$4 \times 5 - 6 = 25 + m$ from ①
$m = -11$

Marks:

i) 2 marks

ii) 2 marks

1 mark – Subst method – removal of one term in simul eqn

1 mark – correct expansion

Could also be done by equating coefficients

$$(x-5)(x^2-x+1)(x-\beta)$$
$$= x^4-(6+b)x^3+6(\beta+1)x^2-(5+6\beta)x+5\beta$$

$\therefore \quad n = 5\beta$

Coefficient of x^2

$$6(\beta+1) = 36$$
$$\beta = 5$$
$$\therefore \quad n = 25$$
$$m = -(6+\beta) = -11$$

---

2 marks

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

1 mark

$$\frac{2x}{a^2}+\frac{2y}{b^2}\frac{dy}{dx}=0$$

$$\frac{dy}{dx} = -\frac{b^2x}{a^2y}$$

$$y-y_1 = -\frac{b^2x_1}{a^2y_1}(x-x_1)$$

$$a^2y_1y-a^2y^2{}_1 = -b^2x_1x+b^2x^2{}_1$$

$$b^2x_1x+a^2y_1y = a^2y^2{}_1+b^2x^2{}_1$$

$\div \quad a^2\beta$

$$\frac{x_1x}{a^2}+\frac{y_1y}{b^2}=\frac{x_1{}^2}{a^2}+\frac{y_1{}^2}{b^2}$$

$$\frac{x_1x}{a^2}+\frac{y_1y}{b^2}=1$$

$$x = a\cos\theta \qquad y = b\sin\theta$$

$$\frac{a\cos\theta x}{a^2}+\frac{b\sin\theta\, y}{b^2}=1$$

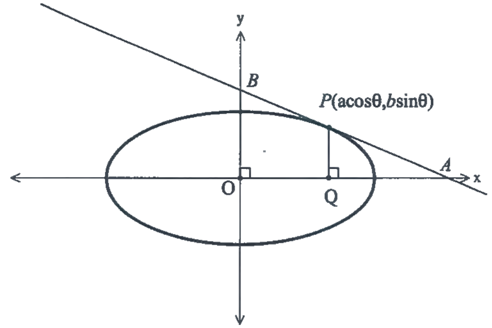
$$\frac{x\cos\theta}{a}+\frac{y\sin\theta}{b}=1$$

at $x = 0$ $y = \dfrac{b}{\sin\theta}$

at $y = 0$ $x = \dfrac{a}{\cos\theta}$

2 marks

1 mark



Ratio of transversals on parallel lines

$$\frac{AP}{AQ}=\frac{PB}{QO}$$

$$\therefore \frac{AP}{PB}=\frac{AQ}{QO}$$

At $y = 0$ (from equation to tangent)

$$x = \frac{a}{\cos\theta}$$

$$\therefore AQ = \frac{a}{\cos\theta} - a\cos\theta = \frac{a(1-\cos^2\theta)}{\cos\theta}$$

$$OQ = a\cos\theta$$

$$\frac{AP}{QO} = \frac{a(1-\cos^2\theta)}{\cos\theta}\times\frac{1}{a\cos\theta} = \frac{\sin^2\theta}{\cos^2\theta}$$

$$= \tan^2\theta$$

$$\frac{PA}{PB} = \tan^2\theta$$

Q13

c-i

String will break above

$T = mg = 40 \times 9.8 = 392N$

$F = mr\omega^2$

$392 = \frac{1}{2} \times 2 \times \omega^2$

$\omega = \sqrt{392} = 14\sqrt{2}$ m/s $\cong$ 19.8

2 marks

c-ii

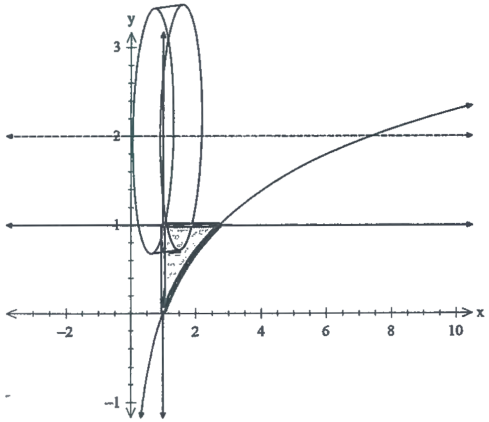
$v = r\omega$

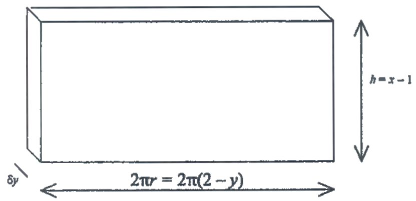
$v = \frac{1}{2} \times 14\sqrt{2}$

$= 7\sqrt{2}$ m/s $= 9.9$

1 mark

13-d





4 marks

3 marks

2 marks

1 mark

- Correct δV

MSC HSC Mathematics X2 Solutions

$$\delta V = 2\pi r.h.w$$
$$= 2\pi(2-y)(x-1)\delta y$$

$$y = \ln x \Rightarrow x = e^y$$

$$\delta V = 2\pi(2-y)(e^y-1)\delta y$$

$$V \cong \lim_{\delta y \to 0} \sum_0^1 2\pi(2-y)(e^y-1)\delta y$$

$$V = 2\pi\int_0^1 (2-y)(e^y-1)dy$$
$$= 2\pi\int_0^1 2e^y - 2 - ye^y + y\, dy$$
$$= 2\pi\left\{\left[2e^y - 2y + \frac{y^2}{2}\right]_0^1 - \int ye^y\, dy\right\}$$

Let $m = ye^y$

$$\frac{dm}{dy} = e^y + ye^y$$

$$\therefore \quad ye^y = \int e^y + ye^y\ dy$$

$$\int ye^y\, dy = ye^y - \int e^y dy = ye^y - e^y + C$$

$$V = 2\pi\left[2e^y - 2y + \frac{y^2}{2} - ye^y + e^y\right]_0^1 = 2\pi$$
$$= 2\pi\left\{\left(3e - 2 + \frac{1}{2} - e\right) - (3-0-0-0)\right\}$$
$$= 2\pi\left(2e - \frac{9}{2}\right)$$
$$= (4e-9)\pi\ u^3$$

Question 14

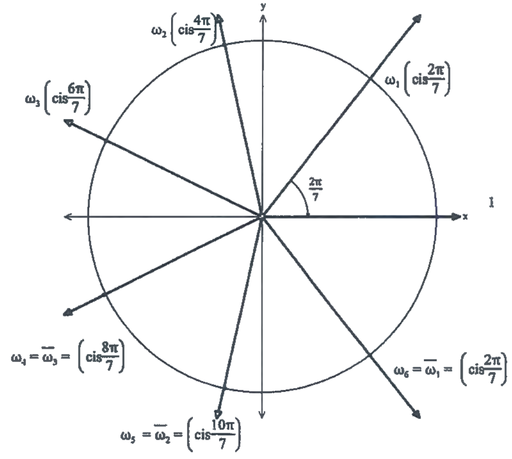
ai	$\frac{dt}{dx} = \sec^2 x$ $= 1 + \tan^2 x$ $= 1 + t^2$ $\therefore \frac{dx}{dt} = \frac{1}{1+t^2}$	1 mark correct solution
aii	$\int\left(\frac{1}{1+\left[\frac{2t}{1+t^2}\right]} \times \left(\frac{dt}{1+t^2}\right)\right)$ $= \int \frac{1+t^2}{1+t^2+2t} \times \frac{dt}{1+t^2}$ $= \int \frac{dt}{(1+t)^2}$ $= \int (1+t)^{-2}\ dt$ $= -(1+t)^{-1} + c$ $= \frac{1}{1+\tan x} + c$	3 marks-correct solution 2 marks- correct substitutions and integration with ONLY one error in correct progress to answer. 1 mark-correct substitions
b	M_{PQ} $= \left(\frac{c(p+q)}{2}, \frac{c}{2}\left(\frac{1}{p}+\frac{1}{q}\right)\right)$ $= \left(\frac{c(p+q)}{2}, \frac{c}{2}\frac{(p+q)}{pq}\right)$ $p - q = 4 \Rightarrow p = 4 + q$ $x = \frac{c}{2}(4+2q)$ $x = c(2+q)$ $\frac{2x}{c} = 2p - 2$ $\frac{x}{c} = 2 = q$ $q = \frac{x}{c} - 2$ $y = \left(\frac{1}{2}\left(\frac{cp+cp}{pq}\right)\right)$ $= \frac{c}{2}\left(\frac{p+q}{pq}\right)$ $= \frac{c}{2}\left(\frac{4+2q}{(4+q)q}\right)$ $= \frac{2c}{2}\left(\frac{2+q}{(4+q)q}\right)$ $= \frac{c\left(\frac{x}{c}\right)}{\left(\frac{x}{c}+2\right)\left(\frac{x}{c}-2\right)}$ $= \frac{x}{\left(\frac{x}{c}\right)^2 - 4}$ $= \frac{x}{\left(\frac{x^2-4c^2}{c^2}\right)}$ $\therefore$ Locus $\Rightarrow y = \frac{xc^2}{x^2-4c^2}$	3 marks- correct solution 2 marks- one error in correct progress to locus using given info without QS 1 mark-one correct use of given info simplify x or y

ci	$r = h\tan\theta$ $T\sin\theta = M(h\tan\theta)\omega^2$ $T\cos\theta = Mg$ $\therefore \tan\theta = \frac{h\tan\theta\omega^2}{g}$ $\omega^2 = \frac{g}{h}$ $\omega = \sqrt{\frac{g}{h}}$ $per\iota d = \frac{2\pi}{\omega}$ $= \frac{2\pi}{\sqrt{\frac{g}{h}}}$ $= 2\pi\sqrt{\frac{h}{g}}$	2 marks-correct solution 1 mark-correct equations for motion
cii	$T\cos\theta = Mg$ $\cos\theta = \frac{Mg}{T}$ $T\sin\theta = Mr\omega^2$ $T = \frac{mr\omega^2}{\sin\theta}$ $\cos\theta = \frac{Mg}{\frac{mr\omega^2}{\sin\theta}}$ $= \frac{Mg\sin\theta}{mr\omega^2}$ $= \frac{g\sin\theta}{r\omega^2}$ $= \frac{g\left(\frac{r}{L}\right)}{r\omega^2}$ since $\sin\theta = \frac{r}{L}$ $= \frac{g}{L\omega^2}$	3 marks- correct solution 2 marks- correct use of equations of motion to create equation in cos and significant relevant progress to expression 1 mark-demonstration of use of equations of motion to obtain required result
ciii	$\cos 2\theta = 2\cos^2\theta - 1$ $\cos 2\theta = \frac{g}{L\mu^2}$ $\therefore \frac{g}{L\mu^2} = 2\left(\frac{g}{L\omega^2}\right)^2 - 1$ from part ii $= \frac{2g^2}{L^2\omega^4} - 1$ $= \frac{2g^2L^2\omega^4}{L^2\omega^4}$ $\therefore \mu^2 = \frac{gL^2\omega^4}{L(2g^2 - L^2\omega^4)}$ $= \frac{gL\omega^4}{2g^2 - L^2\omega^4}$	3 marks- correct solution 2 marks- correct use of both cos equations and significant relevant progress to expression 1 mark-recognise the of use of both cos result to create/equate

Question 15

 Q15

a 1	$\int xe^{2x}\ dx$ $u = x \qquad v' = e^{2x}$ $u' = 1 \qquad v = \frac{1}{2}e^{2x}$ $I = \frac{x}{2}e^{2x} - \frac{1}{2}\int e^{2x}\ dx$ $= \frac{x}{2}e^{2x} - \frac{1}{4}e^{2x} + C$	3 marks – correct solution 2 marks – one error 1 mark – correct separation into parts
a-ii	$\int_0^{\frac{\pi}{2}} \sin\theta\ (1-\cos\theta)^2\ d\theta$ $= \int_0^{\frac{\pi}{2}} \sin\theta\ (1 - 2\cos\theta + \cos^2\theta)\ d\theta$ $= \int_0^{\frac{\pi}{2}} \sin\theta - 2\sin\theta\cos\theta + \sin\theta\cos^2\theta\ \ d\theta$ $= \int_0^{\frac{\pi}{2}} \sin\theta - \sin 2\theta + \sin\theta\cos^2\theta\ \ d\theta$ $= \left[-\cos\theta + \frac{\cos 2\theta}{2} - \frac{\cos^3\theta}{3}\right]_0^{\frac{\pi}{2}}$ $\left[-\cos\frac{\pi}{2} + \frac{\cos\pi}{2} - \frac{\cos\frac{3\pi}{2}}{3}\right] - \left[-\cos 0 + \frac{\cos 0}{2} - \frac{\cos^3 0}{3}\right]$ $= \left[0 - \frac{1}{2} + 0\right] - \left[-1 + \frac{1}{2} - \frac{1}{3}\right]$ $= -\frac{1}{2} + \frac{1}{2} + \frac{1}{3} = \frac{1}{3}$ Better version	3 marks - Correct solution 2 marks - Correct use of trig identity 1 mark - One correct integration -

	let $u = 1 - \cos\theta$ $du = \sin\theta d\theta$ $\theta = \frac{\pi}{2} \Rightarrow u = 1$ $\theta = 0 \Rightarrow u = 0$ $\int_0^1 u^2\ du$ $= \left[\frac{u^3}{3}\right]_0^1 = \frac{1}{3}$	
b-i		2 marks – all solutions correct 1 mark -
b-ii	$P(z) = (z-1)(z-\omega_1)(z-\omega_2)(z-\omega_3)(z-\omega_4)(x-\omega_5)(z-\omega_6)$ $P(z) = (z-1)(z-\omega_1)(z-\omega_6)(z-\omega_2)(z-\omega_5)(z-\omega_3)(z-\omega_4)$ $= (z-1)(z-\omega_1)\left(z-\overline{\omega_1}\right)(z-\omega_2)\left(z-\overline{\omega_2}\right)(z-\omega_3)\left(z-\overline{\omega_3}\right)$ $= (z-1)\left(z^2 - 2Re(\omega_1)z + (\lvert\omega_1\rvert)^2\right)\left(x^2 - 2Re(\omega_2)z + (\lvert\omega_2\rvert)^2\right)\left(x^2 - 2Re(\omega_3)z\right.$ $= (z-1)\left(z^2 - 2\cos\frac{2\pi}{7}\theta z + 1\right)\left(z^2 - 2\cos\frac{4\pi}{7}\theta z + 1\right)\left(z^2 - 2\cos\frac{6\pi}{7}\theta z + 1\right)$	
	2marks 1 mark	

$$\Sigma\alpha = -\frac{b}{a} = \frac{0}{1}$$

$$1 + \text{cis}\frac{2\pi}{7} + \text{cis}\frac{4\pi}{7} + \text{cis}\frac{6\pi}{7} + \text{cis}\frac{8\pi}{7} + \text{cis}\frac{10\pi}{7} + \text{cis}\frac{12\pi}{7} = 0$$

$\therefore$

$$\cos\frac{2\pi}{7} + \cos\frac{4\pi}{7} + \cos\frac{6\pi}{7} + \cos\frac{8\pi}{7} + \cos\frac{10\pi}{7} + \cos\frac{12\pi}{7} = -1$$

$$\cos\frac{2\pi}{7} + \cos\frac{4\pi}{7} + \cos\frac{6\pi}{7} + \cos -\frac{6\pi}{7} + \cos -\frac{4\pi}{7} + \cos -\frac{2\pi}{7} = -1$$

as $\cos x = \cos(-x)$

C

$$\cos\frac{2\pi}{7} + \cos\frac{4\pi}{7} + \cos\frac{6\pi}{7} + \cos\frac{6\pi}{7} + \cos\frac{4\pi}{7} + \cos\frac{2\pi}{7} = -1$$

$$2\left(\cos\frac{2\pi}{7} + \cos\frac{4\pi}{7} + \cos\frac{6\pi}{7}\right) = -1$$

$$\cos\frac{2\pi}{7} + \cos\frac{4\pi}{7} + \cos\frac{6\pi}{7} = -\frac{1}{2}$$

as $\cos(\pi - x) = -\cos x$

$$-\cos\frac{5\pi}{7} - \cos\frac{3\pi}{7} - \cos\frac{\pi}{7} = -\frac{1}{2}$$

$$\cos\frac{5\pi}{7} + \cos\frac{3\pi}{7} + \cos\frac{\pi}{7} = \frac{1}{2}$$

~~2 marks~~
1 mark

~~3/4 marks - correct solution~~
~~2 - mark -~~

2 - marks - correct solution

1 - mark - one error

Asymptotes $\quad y = \pm\frac{b}{a}x \qquad m_{PR} = -\frac{a}{b} \quad m_{PQ} = \frac{a}{b}$

$\therefore$ equations are

$$bx - ay = 0$$
$$bx + ay = 0$$

PR and PQ are perpendicular distances.

$$dist = \frac{|Ax_1 + By_1 + C|}{\sqrt{a^2 + b^2}}$$

$$PR = \frac{|bx_1 + ay_1|}{\sqrt{a^2 + b^2}}$$

$$PQ = PR = \frac{|bx_1 - ay_1|}{\sqrt{a^2 + b^2}}$$

$$PR.PQ = \frac{|bx_1 + ay_1|}{\sqrt{a^2 + b^2}} \times \frac{|bx_1 - ay_1|}{\sqrt{a^2 + b^2}}$$
$$= \frac{\left|b^2x^2{}_1 - a^2y^2{}_1\right|}{(a^2 + b^2)}$$

$$e^2 = 1 + \frac{b^2}{a^2} \Rightarrow a^2e^2 = a^2 + b^2$$

$$\frac{x^2{}_1}{a^2} - \frac{y^2{}_1}{b^2} = 1$$
$$b^2x^2{}_1 - a^2x_2 = a^2b^2$$

$$PQ.PR = \frac{a^2b^2}{a^2e^2} = \frac{b^2}{a^2}$$

2 marks

Denominator formed correctly as e^2

1 mark – correct expression for both distances

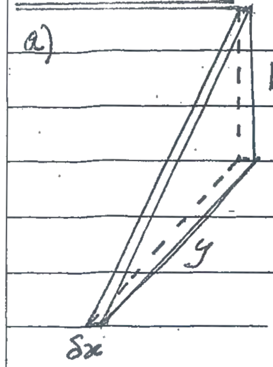
3 marks

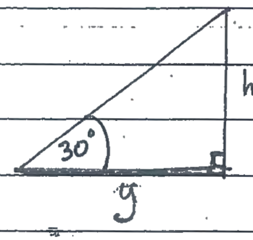
2 marks

1 mark

Question 16 | Marks

a)





$h = y\tan 30^\circ = \frac{1}{\sqrt{3}}y$

$A = \frac{1}{2}yh = \frac{1}{2\sqrt{3}}y^2$

$\delta V = A\delta x \Rightarrow \delta V = \frac{1}{2\sqrt{3}}y^2\,\delta x$

$V = \int_{-5}^{5} \frac{1}{2\sqrt{3}}y^2\,dx = 2 \times \frac{1}{2\sqrt{3}}\int_0^5 y^2\,dx$

$y^2 = 25 - x^2 \Rightarrow V = \frac{1}{\sqrt{3}}\int_0^5 (25 - x^2)\,dx$

$= \frac{1}{\sqrt{3}}\left[25x - \frac{x^3}{3}\right]_0^5$

$V = \frac{1}{\sqrt{3}}\left[\left(125 - \frac{125}{3}\right) - 0\right]$

$V = \frac{250}{3\sqrt{3}} = \frac{250\sqrt{3}}{9}\ u^3$

Marks

3 Marks: Correct solution

2 Marks: Obtains a correct volume element AND correct primitive function

1 Mark: Obtains a correct volume element

b) (i)

$x = \tan\theta$

$dx = \sec^2\theta\,d\theta$

$x = 0 \Rightarrow \theta = 0$

$x = 1 \Rightarrow \theta = \pi/4$

$\int \frac{\log_e(1+x)}{1+x^2}\,dx = \int \frac{\log_e(1+\tan\theta).\sec^2\theta\,d\theta}{1+\tan^2\theta}$

$= \int \frac{\log_e(1+\tan\theta)\sec^2\theta\,d\theta}{\sec^2\theta}$

$= \int \log_e(1+\tan\theta)\,d\theta$

Marks

1 Mark: Correct solution with sufficient working out provided.

(ii) $I = \int_0^{\pi/4} \log_e(1+\tan\theta)\,d\theta = \int_0^{\pi/4} \log_e\left\{1+\tan\left(\frac{\pi}{4}-\theta\right)\right\}d\theta$

$1 + \tan\left(\frac{\pi}{4}-\theta\right) = 1 + \frac{\tan\frac{\pi}{4} - \tan\theta}{1+\tan\frac{\pi}{4}\tan\theta}$

$= 1 + \frac{1-\tan\theta}{1+\tan\theta}$

$= \frac{1+\tan\theta+1-\tan\theta}{1+\tan\theta}$

$= \frac{2}{1+\tan\theta}$

$I = \int_0^{\pi/4} \log_e\left\{\frac{2}{1+\tan\theta}\right\}d\theta$

$= \int_0^{\pi/4} \log_e 2\,d\theta - \int_0^{\pi/4} \log_e(1+\tan\theta)\,d\theta$

$I = \left[(\log_e 2)\theta\right]_0^{\frac{\pi}{4}} - I$

Marks

3 Marks: Correct solution

2 Marks: Attempts to use $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$

1 Mark: Attempts to use integration by parts and obtains a correct relevant expression

	Marks
$2I = \frac{\pi}{4}\log_e 2 - 0$	
$I = \frac{\pi}{8}\log_e 2$	
$\int_0^1 \frac{\log_e(1+x)}{1+x^2}\,dx = \frac{\pi}{8}\log_e 2$	
(c) (i) $\int \frac{x}{(1+x^2)^n}\,dx = \int x(1+x^2)^{-n}\,dx = \frac{1}{2}\int 2x(1+x^2)^{-n}\,dx$	2 Marks Correct solution with sufficient working
$= \frac{1}{2}\left\{\frac{(1+x^2)^{-n+1}}{-n+1}\right\} = \frac{1}{2}\left\{\frac{(1+x^2)^{-(n-1)}}{-(n-1)}\right\}$	
$= \frac{-1}{2(n-1)}\left\{\frac{1}{(1+x^2)^{n-1}}\right\} + C$	1 Mark Obtains a correct reverse chain rule integral expression.
(ii) $I_n = \int_0^1 \frac{1}{(1+x^2)^n}\,dx = \int_0^1 \frac{1+x^2-x^2}{(1+x^2)^n}\,dx$	3 Marks Correct solution
$= \int_0^1 \frac{1+x^2}{(1+x^2)^n}\,dx - \int_0^1 \frac{x^2}{(1+x^2)^n}\,dx$	
$= \int_0^1 \frac{1}{(1+x^2)^{n-1}}\,dx - \int_0^1 x^2(1+x^2)^{-n}\,dx$	2 Marks Obtains expressions ① and ②
$I_n = I_{n-1} - \int_0^1 x^2(1+x^2)^{-n}\,dx$ ①	
But $\int_0^1 x^2(1+x^2)^{-n}\,dx = \int_0^1 x\left\{x(1+x^2)^{-n}\right\}dx$	1 Mark Obtains expression ①
$u = x \Rightarrow u' = 1$	
$dv = x(1+x^2)^{-n} \Rightarrow v = \int x(1+x^2)^{-n}\,dx = \frac{1}{2(n-1)}\left\{\frac{1}{(1+x^2)^{n-1}}\right\}$	

	Marks
$\int_0^1 x^2(1+x^2)^{-n}\,dx = \left[\frac{-x}{2(n-1)(1+x^2)^{n-1}}\right]_0^1 - \int_0^1 \left\{\frac{-1}{2(n-1)(1+x^2)^{n-1}}\right\}dx$	
$= \frac{-1}{2(n-1)(2^{n-1})} - 0 + \frac{1}{2(n-1)} I_{n-1}$	
$= \frac{-1}{2^n(n-1)} + \frac{1}{2(n-1)} I_{n-1}$ ②	
$I_n = I_{n-1} - \left\{\frac{-1}{2^n(n-1)} + \frac{1}{2(n-1)} I_{n-1}\right\}$	
$I_n = I_{n-1} + \frac{1}{2^n(n-1)} - \frac{1}{2(n-1)} I_{n-1}$	
$= I_{n-1} - \frac{1}{2(n-1)} I_{n-1} + \frac{1}{2^n(n-1)}$	
$= I_{n-1}\left\{1 - \frac{1}{2(n-1)}\right\} + \frac{1}{2^n(n-1)}$	
$= \left\{\frac{2n-2-1}{2(n-1)}\right\} I_{n-1} + \frac{1}{2^n(n-1)}$	
$I_n = \left\{\frac{2n-3}{2(n-1)}\right\} I_{n-1} + \frac{1}{2^n(n-1)}$	
(iii) $0 \le x \le 1 \Rightarrow (1+x^2) \ge 0$ and $(1+x^2) \le (1+1^2) \le 2$	2 Marks Correct solution
$\therefore$	
$0 \le x \le 1 \Rightarrow 0 < (1+x^2)^n \le 2^n$	

Marks

$(1+x^2)^n \leq 2^n \Rightarrow \frac{1}{1+x^2} \geq \frac{1}{2^n}$

1 Mark

$$\int_0^1 \frac{1}{1+x^2}\,dx \geq \int_0^1 \frac{1}{2^n}\,dx$$

Sufficient relevant progress using a valid method.

$$\geq \frac{1}{2^n}\left[x\right]_0^1$$

$$I_n \geq \frac{1}{2^n}(1-0) \geq \frac{1}{2^n}$$